研究ノート

Drag force from the String Theory in $AdS_5 \times S^5$

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概要

強い相互作用におけるクォーク・グルーオン・プラズマ(QGP) に関する実験結果を，超弦理論を用いて分析を試みる。摂動論的量子色力学(QCD) や格子ゲージ理論が適用できない強結合領域での現象が，超弦理論の双対性を用いて解析可能であると予想されている。これによると，強結合領域は5 次元多様体にコンパクト化した超弦理論に対応すると考えられる。ここでは，超弦理論を5 次元の反ド・ジッター (AdS) 空間にコンパクト化した理論の作用を用いて，プラズマの状態に存在する引力を導く議論を紹介し，これを踏まえて，別のコンパクト化された空間についての議論を行う。

Abstract

We try to analyze the experimental results on the quark-gluon plasma(QGP) in the strong interaction by using the string theory. It is expected that phenomena, that the perturbative quantum chromodynamics(QCD) or the lattice gauge theory cannot be applied to, can be analyzed by using the duality of the superstring theory. According to this the strong coupling region corresponds to the superstring theory compactified on a five-dimensional manifold. In this paper we review the discussion on deriving the drag force in the plasma state from the action of the superstring theory compactified on five-dimensional anti de Sitter(AdS) space. We also discuss another compactified space.

1. Introduction

The string theory has been expected to explain all the four interactions in Nature in one theory, and successful to show some evidence from a theoretical point of view. The four interactions are electromagnetic, weak, strong and gravitational interactions. Gravity always caused a problem in unifying all the interactions*. The string theory has been attractive as it includes all the features of gravity as well as of other three interactions in one. It has been successful as for the completeness of the theory for the Nature, but agreement with experimental facts has been searched for a long time.

On the other hand, physics implied by experiments has been studied both in theoretically and in phenomenologically. A remarkable achievement is a numerical simulation in a study of the strong interaction: there are a couple of on-going experiments,a technique for analyzing supersymmetry is achieved in the lattice gauge theory, and the power of computers is being increased day by day. In the weak coupling region the lattice gauge theory and the analysis by quantum chromodynamics (QCD) are quite successful.

One of the experimental results being obtained is from the Relativistic Heavy Ion Collider (RHIC) in the Brookhaven National Laboratory (BNL), New York,U.S.A. [4, 5]. It is an experiment of studying behaviors of colliding nuclei of Au.The collision is in the strong coupling region according to the data fitting. The strong interaction theory has been successful for problems like high energy collision of electrons and protons, for which the coupling becomes weak due to the asymptotic freedom and the perturbative calculations are available. However, the analysis of the data at RHIC showed the situation is totally different from the previous expectation.It also showed that escaped particles from the beam called the jet become slower as they go through the beam [6]. This energy loss of the jets is difficult to understand from the perturbative picture.

The last decade has been the era of studying the string duality. Especially the

*To see more, see [1, 2, 3] and references therein.

duality between the strong coupling region and the weak coupling region, or the duality between the gravity and the field theory has been studied extensively.We apply this duality to the above new phenomena since it is an approach that can treat non-perturbative effects. We first summarize the string theory literary in the following section, describe what is the strong interaction, and explain the experiment of RHIC. Then, we come back to the string duality and explain the drag force. Finally, we will briefly mention our conjecture using the idea of the D-brane configuration with colors and flavors [7, 8, 9] on the AdS space.

2. The dungeons of the string theory

The string theory had three remarkable evolution eras in its history so far, and we are now in the third stage. The string theory was first developed in an attempt to construct a model of particles with an inner structure. It is described by a twodimensional field theory on a worldsheet with not only the time direction but also the space direction, and therefore has a string picture. It actually describes higher spin states without anomaly. However, the spacetime dimension must be twenty-six for the bosonic string theory and ten for the supersymmetric string theory. The supersymmetry is a symmetry between two kinds of particles, bosons and fermions.

Although the string theory requires either twenty-six or ten dimensions, we know that our spacetime looks only four-dimensional at least for the present possible energy scale. It is necessary to know how to reduce the spacetime dimensions in the string theory. Thus, the second development of the string theory was how we derive an effective field theory in four dimensions. It is called the compactification that gives a reduced spacetime, and the geometry of the compactified manifold in the extra dimensions has an important role in the effective theory. The geometry of the extra dimension determines the gauge group of the effective field theory. The effective field theories in problem should have the same gauge group as the electroweak theory or QCD. Compactified manifolds can be orbifolds, torus, Kähler manifolds or Calabi-Yau (CY) manifolds. This direction of research brought the non-perturbative analysis for the $\mathcal{N} = 2$ super Yan–Mills theory, with a lot of duality relations for effective field theories. At the same time the moduli space of the string theory was studied, and the conformal field theory (CFT) was well developed.

The third development of the string theory has started with the discovery of non-perturbative objects in the string theory. The non-perturbative objects called "D-branes" were originally found from a relation between open strings with the Dirichletboundary condition and the Neumann boundary condition. Introduction of the D-branes brought a drastic revolution to the string theory and field theories. All the effective field theories are interpreted by piling up D-branes, and the gauge groups are represented by the number of D-branes. At some specific situation the geometry of the D-branes generate around them is found, and therefore the gravity on that geometry is also investigated.

The D-brane has the T-duality, which can change the dimension easily, and the web of the string theories have been established, as well as the duality among compactifications of the spacetime. The study of D-branes itself opened yet another dimension to the spacetime of the supersymmetric string theory. To derive four-dimensional spacetime from the type II superstring, which has two sets of supersymmetries, D0-branes for the type IIA string theory and D(-1)-branes for the type IIB string theory have been considered. The action is called the Matrix Model. The matrix model for the type IIA superstring showed that all the five string theories can be treated in one theory, called the M-theory in eleven-dimensional spacetime.

Recently, according to the development in differential geometry a compactification on non-compact manifolds is also defined, and the $\mathcal{N} = 1$ gauge field theory is revealed by a compactification of the M-theory to the G_2 manifold. It is related to the flux compactification, where the flux can sustain the space to be stable. The geometry generated by D-branes is also studied focusing on the counting of the string micro states. This gives the black hole entropy, which was previously studied invoking the idea of the thermodynamics in general relativity.

The survey of field theories and finding soliton solutions were any more independent research after the conjecture on a relation between the gravity induced by the D-branes and the effective field theory on the D-branes. And also the string theory attracts more attention for more realistic physics after the discovery of the relation. The first example of such a relation was the AdS/CFT correspondence, which is the most beautiful correspondence in the theoretical approach[10, 11, 12]*. It stands for the specific case: the geometry is anti de Sitter space (AdS) and the corresponding effective field theory is a conformal field theory with infinite number of colors and SU(4) R-symmetry. In four-dimensional spacetime the theory is called the super Yang–Mills (SYM) theory. CFT is not a favorable theory from a phenomenological point of view, but the conjecture itself was supported by much evidence for this case. CFT has a feature that the coupling in the theory never runs from the weak to the strong, whereas QCD runs and especially shows the confinement and the asymptotic freedom. After the proposal of the AdS/CFT conjecture and the following works supporting the conjecture by concrete examples, most of the interests by physicists including experimentalists have moved to explanations of phenomena in realistic models brought by the AdS/CFT correspondence. Some of the field theorists are interested in the analysis of a field theory coupled to SYM to achieve the fluid dynamics for example. Independently the extension of the AdS/CFT correspondence to the generic case has been searched for, like a generalization of the geometry from AdS to others.

To find a realistic model from the string theory yields other possibility to answer the long standing questions in the effective field theories: why the number of the generations is three, what the origin of mass is, what the dark matter is, how we believe the supersymmetry, and how it is broken if it exists. All of these questions have not been answered by ordinary field theories†.

On the other hand, the experiments in the hadronic collider showed phenomena, which are hard to be explained by the perturbative approach of the field theory, although it explained successfully most of the phenomena in the weak coupling region. Some string theorists claim the phenomena in the quark-gluon plasma is brought by

* To see more, see [1, 2, 3] and references therein.

† Like the special relativity it might happen that the theory that brings the idea never answers the problem; we still do not answer to the existence of the ether.

the string theory, as the data of the plasma is analyzed in the strong coupling region. Therefore, field theorists who study higher loop corrections or the lattice gauge theory focus now on the conjecture. Therefore the string theory is actively being studied to investigate the real physics.

3. Introduction to hadron physics

Hadrons such as neutrons and protons were found after the discovery of photons and electrons. Hadrons are a class of particles made of quarks. Three quark states are called baryons and two quark states as mesons. Electrons and neutrinos are classified as leptons as they are lighter than hadrons. Leptons do not have any further fine structure like hadrons. Leptons interact via the weak and electromagnetic interactions, while hadrons via the strong interaction in addition to them.

The electromagnetic interaction has been familiar since the Ancient Greek, and the gravity since the discovery by Newton after the invention of telescopes. The weak interaction and the strong interaction, however, were found only by the last century after the radiation from radioactive matters was found.

There are three kinds of radiations according to emitted particles: the α-particle which is known to be a helium nuclei, the β -particle which is an electron, and the γ -particle which is a photon. The photon was discovered in experiments such as the photo-electric effect and is a particle representing electromagnetic wave or light. Still the words like α , β and γ -decay are used in the nuclear physics even after all the emitted particles were identified.

3.1 Discovery of the structure of atom

Let us see first how we detect particles inside of matters. Informations of those particles are found by looking at signals coming from a collision process, which occurs by interactions between accelerated incident particles and a target matter. Among the four interactions the gravitational interaction is the weakest, the weak interaction is the second, the electromagnetic interaction is the third, and the strong interaction is the most strongest at the energy scale of the present accelerators.

The inner structure of atoms was found by collider experiments: it is seen by colliding electrons to Au nuclei in a thinly spread gold sheet. The result of the scattering of electrons indicated that a positively charged particle, which cancels the negative charge of the electrons in the atom, is located only at the center of the atom. Such a model of the structure of atoms is called the Rutherford model. It is based on the following facts. Electron is known to be a much lighter particle than a nuclei. The electromagnetic interaction is working to the nuclei and electrons around the nuclei. By observing the scattering angle of incident electrons, scattered by the electrons and the nuclei in the atom, the inner structure of the atom was found. The result indicated that the atom has the structure of electrons moving along orbits far away from the nuclei at the center of the atom, rather than a mixture of electrons and nuclei.

A hydrogen atom contains an electron and a positively charged particle called a proton. The mass of a proton is

$$m_p = 938.272 \quad \mathrm{MeV}/c^2 \tag{3.1}$$

Neutron is a electrically neutral particle. The mass of a neutron is

$$m_n = 939.566 \quad \mathrm{MeV}/c^2 \tag{3.2}$$

Nuclei other than the hydrogen are bound states of protons and neutrons, which are collectively called nucleons.

By the study of chemical interactions in the development of the chemistry orbits of electrons around a nuclei have been discovered. However, a question has appeared: what interaction binds nucleons to form nuclei against repulsive electromagnetic interactions among protons? Therefore we discovered there exists some interaction which only works at short distance, is stronger than the electromagnetism, and further works to protons and neutrons at the same time to glue them into a nuclei. As it is stronger than other interactions, this interaction is called the strong interaction.

3.2 Quark model

Nowadays the strong interaction is well described by a quantum field theory called quantum chromodynamics or QCD in short. This theory of the strong interaction explains behaviors of hadrons. Hadrons are particles subject to the strong interaction and are divided into two types: mesons and baryons. Baryons are particles with half-integer spins. Protons and neutrons are baryons. Mesons are particles with integer spins. Pion is a meson. These particles are composite states made of two or three quarks. Various combinations of different types of quarks give various hadrons.

Quarks were suggested from a number of decay processes of unstable hadrons. A couple of discoveries of (partially) conserved quantities such as the isospin, baryon number and strangeness (or hypercharge, equal to the sum of isospin and strangeness) brought the idea of the quark model that hadrons are made of more fundamental particles called quarks. At present six kinds of quarks are known. These six quarks are distinguished by the "flavor" u, d, c, s, t, b. Flavors of quarks are not changed in the strong interaction process. On the other hand there is an interaction called the weak interaction which changes flavors. It work also to leptons. The weak interaction was found in the neutron β-decay, by which neutrino was discovered. The weak interaction will not be explained in the present paper.

3.3 Deep Inelastic Scattering

Scattering process in particle physics looks a bit like a black box problem. We know the target nucleon and the incident particle, and we observe the energy and the direction of the scattered particle. We do not know about the process during the scattering because of the limitation of quantum mechanics.

Deep inelastic scattering is different from elastic scattering in which only energy and momentum of the particles are changed; it changes states of the particles after the scattering as well besides its motion. Some of them keep the kind of the parti cles and only changes them to excited states. Some others produce many hadrons instead. The final state of the target nucleon is various enough, and we have to count all the states in

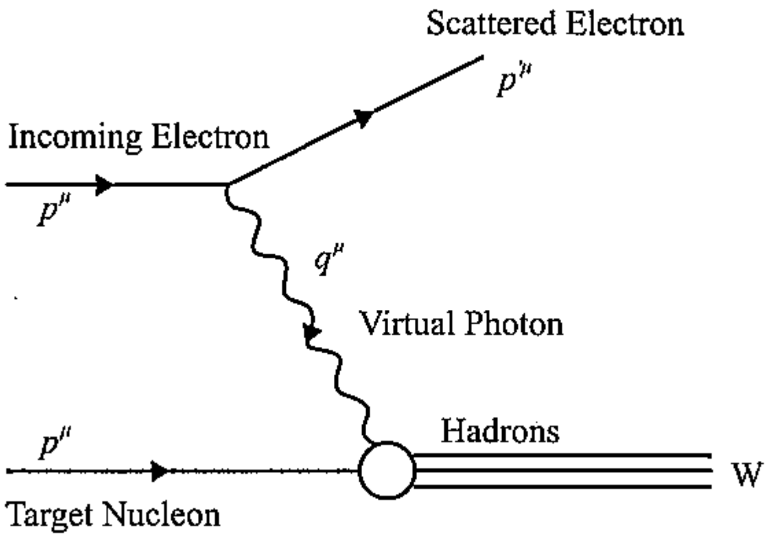

Figure 1: Feynman diagram of the deep inelastic scattering

the black box according to the quantum dynamics.

At low energy collision these are mainly one photon process in Fig. 1. In this diagram we denote the four-dimensional momentum p^μ for the incoming electron, P^μ for the target nucleon, p'^μ for the scattered electron, q^μ for the virtual photon, and the effective mass W for the produced hadron with

$$p^\mu = (\sqrt{p^2 + m^2c^2}, 0, 0, p), \tag{3.3}$$

$$p^\mu = (Mc, 0, 0, 0). \tag{3.4}$$

Here we have denoted the indices of four-dimensional spacetime as μ, $\nu = (t, x, y, z)$.

According to the experimental data of collisions by the accelerator, all the hadrons are composite quark states. Mesons are combinations of quark and antiquark, while baryons are three quark states.

3.4 Hadron jet process

Accelerators are constructed in various places in the world, for verifying theoretical predictions at each energy level and for obtaining more results which are key to new physics. Linear accelerators are easy to accelerate particles and are suitable for experiments of higher energy processes. Circular accelerators use a booster to accelerate particles in the tunnel of magnetic field to high energy, and collide them to particles accelerated in the opposite direction in the detector region. This circular system is called cyclotron, synchrotron or synchrocyclotron according to the technics for the acceleration.

What do we observe at the accelerators? We observe the energy of scattered electrons and scattered angle of the jet. The Feynman diagram in Fig. 2 shows the process with the time evolving from the left to the right, where the particle is identified from the measurements of energy and the angle at which the heat comes from the source. The experimental data consists of the energy of incoming and outgoing particles and the scattering cross section.

In this process there appears a virtual photon in an intermediate state. We apply the uncertainty principle to this photon. According to the uncertainty principle the product of uncertainties of two independent variables in the phase space of the Hamiltonian formalism is always greater than $\hbar$. For example, if we observe the position of a particle with an error Δx, we only observe the momentum with an error Δp satisfying

$$\Delta x \Delta p \geq \hbar, \tag{3.5}$$

where ħ is the quantum unit of the action defined from the Planck constant h as

$$\hbar = \frac{h}{2\pi}. \tag{3.6}$$

Or, if we observe the decay time with an error Δt, we only observe its energy with an error $\hbar$ satisfying

$$\Delta t \Delta E \geq \hbar. \tag{3.7}$$

Here we see the existence of the photon, therefore it is impossible to know the energy, meaning that its mass is not necessarily vanishing. As the photon is massive, so there exists the longitudinal mode as well as the transverse modes. So the total cross section of the scattering between the imaginary photon and the nucleon is formed with the ones for both direction. The total cross section for each particle is related to the

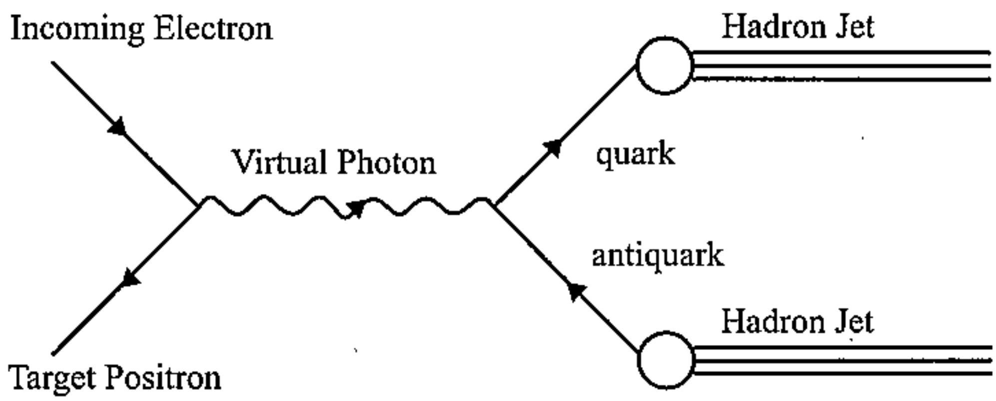

Figure 2: Feynman diagram of the jet process

structure function. The present process is consists of the observation quantities E, E', θ_{Lab}, energy of incoming electron, energy of the scattered electron and the scattering angle. The total cross section should depend on the function of the solid angle and the energy of the final state. The differential cross section with these variable is the desirable quantity to discuss the physics. To treat this kind of quantum mechanics we use the quantum field theory. In the quantum field theory the dimension of the theory is measured by the mass dimension. And we often scale the observed quantity to dimensionless.

The structure functions of nucleons probed by photons depend on the difference of the energies of the incoming electron and the outgoing electron, and the squared four-momentum of the virtual photon:

$$\nu \equiv \frac{P^\mu q_\mu}{M} = E - E', \quad Q^2 \equiv -q^\mu q_\mu = 4\frac{EE'}{c^2}\sin^2\left(\frac{\theta_{Lab}}{2}\right). \tag{3.8}$$

The conservation law of the scattering process yields

$$\left(\frac{W}{c^2}\right) \equiv (P^\mu + q^\mu)^2 = (P^\mu + p^\mu - p'^\mu)^2 = (Mc)^2 - Q^2 + 2M\nu, \tag{3.9}$$

where W is the effective mass of the hadron.

If we take the dimensionless variable

$$x \equiv \frac{Q^2}{2M\nu}, \tag{3.10}$$

the two structure functions depends only on x at high energy $Q^2 \geq 1$ GeV/c^2, $W \geq 2$ GeV:

$$Mc^2 W_1(Q^2, \nu) \sim F_1(x), \quad \nu W_2(Q^2, \nu) \sim F_2(x). \tag{3.11}$$

The cross section for the deep inelastic scattering is obtained as

$$\frac{d^2\sigma}{dxdy} = \left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^2 \frac{\pi\hbar^2}{EM} \frac{1}{x^2y^2}\left[2\,(1-y)\,F_2(x) + 2y^2xF(x)\right], \tag{3.12}$$

where ϵ_0 is the vacuum permittivity and the variable y is given by

$$y \equiv \frac{\nu}{E} = \frac{E-E'}{E}. \tag{3.13}$$

In the parton model with large enough P a quark in the target nucleon has four momentum $P^\mu \sim (P, 0, 0, P)$. The energy-momentum conservation law with the longitudinal momentum ξP^μ yields

$$0 = (\xi P^\mu + q^\mu)(\xi P_\mu + q_\mu) \sim 2\xi M\nu - Q^2. \tag{3.14}$$

Therefore, ξcan be identified with the scaling variable x. We conclude that the point-like parton is identified with the quark with a half-integer spin, as the structure functions are the delta function of ν. If the parton is a Dirac particle, the two structure functions satisfy the relation

$$F_2(x) = 2xF_1(x), \tag{3.15}$$

which actually agrees with the experimental result. The parton model was also checked by the neutrino scattering process, and the existence of the quark was established.

The data of the electron-positron scattering showed that quark has three internal degrees of freedom, which is now called colors. Actually these degrees of freedom are responsible for the strong interaction. Gluons couple to them as in the same way photons couple to electric charges. There are six kinds of quarks in all, which are distinguished by flavors: up, down, charm, strange, to and bottom. Each quark with the electric charge $\frac{2}{3}$ is paired with a quark with the electric charge $-\frac{1}{3}$. The first pair is the up quark and the down quark, which constitute nucleons. The second and the third pairs are the charm and the strange, and the top and the bottom. These three pairs belong to the first, the second and the third generations, respectively. The weak interaction actually mixes the generations. The generation mixing is being studied, for instance, by a neutrino experiment at the neutrino detector, Super Kamiokande, Japan.

4. Recombination of quarks in the history of Universe

Hadrons are combinations of quarks, and the gluon enters to the process of scattering. By the strong interaction it is difficult to take out quarks from hadrons. However, at higher energy scale it is believed that quarks can exist not only as hadrons but also as as plasma. Plasma of quarks means a phase of free quarks and gluons. It existed at a certain era at the beginning of the universe, so-called the radiation dominated era of the big-bang cosmology. According to the Big-bang theory the temperature of the universe gets colder and colder, and the electromagnetic interaction rarely occurs. The end of the electromagnetic interaction process yields the hadronic phase of the universe.

The phase transition happened everywhere in the universe, as the universe is homogeneous and isotropic. A number of small bubbles of the hadronic phase expanded, and finally the hadronic phase fill everywhere. Perhaps a few regions in the quark phase could survive in the hadron gas finally. These regions in the quark phase have been thought to be filled by a gas of strange quarks, and it is conjectured that they disappear by the evaporation at the temperature of 50 MeV to 1 MeV. However, neutrons can expand to everywhere while proton stay around the bubbles by the evaporation, as neutrons have a longer mean free path than electrons. This phenomena is called "the era of recombination", where the Big–bang universe is cleared up from the cloud of QGP.

The conclusion is that the ratio of the densities of neutrons and protons in the space is not homogeneous enough. It conflicts with the assumption of the standard model of the particle physics, in which it is successful to describe the field theoretical interactions in one theory. According to the discussion of the baryon density of the universe $\Omega_B \sim 1$, and it is actually seems true by the recent observation. However it is hard to explain the measurement ratio of Lithium and Helium.

Although the possibility of the existence of strange matter in the big–bang model is eliminated, aside the name of the QGP*, the behavior of QGP at experiments looks totally different from the free gas model as was thought just like above. QGP is theoretically guaranteed by the Bjorken picture and higher loop simulations of the quark sea with the chemical potential as they fit to the data of the experiments in the region of the weak coupling. However, at the experiment of RHIC new phenomena in the strong coupling region were discovered. An interpretation of them are needed.

4.1 Purpose of the Experiment at RHIC

To know about properties of QGP itself, like the control effect of higher transverse momentum and the phase transition to the hadron phase, it is necessary to observe the state of QGP at experiments. It should be seen by the nucleon–nucleon collision at high energy. It is also observed as the fire ball effect of the cosmic ray, but it is better to have the state at laboratory.

Relativistic Heavy Ion Collider (RHIC) in the Brookhaven National Laboratory (BNL), U.S.A, was founded for that purpose. QGP was actually well observed, and it showed the phase behaves as a perfect fluid rather than a fluid with viscosity predicted

*For more detailed discussions, see [4, 5].

by the perturbative QCD in 2006 April. The perturbative QCD describes the asymptotic freedom, the property that the effective coupling constant vanishes in the high energy limit. At limited energy it also predicts a correction and this result agrees with the experiments.

On the other hand, to find qualitative behavior of the confinement, a nonperturbative treatment is required. So the qualitative picture of the confinement is well understood by the lattice gauge theory, and simulated by the numerical calculations. Analytically, quarks are confined in hadrons as the color gluon has the asymptotic freedom. The confinement is discussed in a similar way to the Meissner effect in the superconductivity, and even in the condensed matter physics, the fractional charge is found.

The discovery of the suppressed (π^0-)production at large transverse momentum in central Au–Au collisions [14] has not explained by the theory.

4.2 Fireball from the nucleon-nucleon scattering

Even in the electron–positron collisions there exists a boosted nucleon forward and backward at the center of mass system, which is called a leading particle. Therefore many baryons in nuclei just pass by the target nucleon. The heat energy is only from the released energy by the deceleration of the nucleon.

Second, although the width of the nuclei vanishes by the Lorentz contraction, the experiment tells that slow quarks are produced inside of the proton. Therefore the uncertainty principle tells that the width never vanishes, and according to the experiment the width is kept to be 1×10^{-15} m or 1 fm.

QGP should have enough energy density and local thermal equilibrium states, and it is possible to describe the fireball by a perfect fluid dynamics. The perturbative QCD predicts that QGP is a free gas of quarks and gluons. However the experimental data showed an enormous number of unexpected trajectories, which happens to a fluid. Fluid dynamics for a perfect fluid should explain the behavior of QGP.

On the other hand, other experiment already showed that the states of QGP is beyond the frame of the perturbative QCD. By the jet quenching gluons and hadrons lose their speed while crossing the fireball. So there exists some force dragging a jet, and it is expected that the QGP fluid is denser than the nucleus sea we have observed.

Since the large momentum suppression happens anyways, a certain drag force should exist. But the problem is that to derive it we need to study the strong coupling region of the theory; all the known approaches to describe QGP so far was the effective theory in the weak coupling region. Drag force is therefore discussed not by the field theorists but by the string theorists, as the string theory is expected to describe the effective field theory in the strong coupling region according to the AdS/CFT conjecture.

5. String theoretical approach to QGP

Here we will describe the string theory on AdS_5 space and its dual field theory. If we admit the supersymmetry, a symmetry between fermions and bosons, the string theory gives the $\mathcal{N} = 4$ supersymmetry to the dual CFT. In four dimensions the most well-known CFT is the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory. String theory on AdS_5 space has the D-brane picture originally. The D-brane is a non-perturbative object in the string theory. It corresponds to a soliton in an effective gravitational theory. If the number of D-branes is large enough, and if one looks at its geometry near the horizon, the geometry looks like AdS space. We obtain the SYM as a dual field theory in the same limit.

5.1 $\mathcal{N} = 4$ SYM theory and QGP

Using the AdS/CFT conjecture hydrodynamics features of $N = 4$ SYM have been studied by field theorists [15, 16]. In the four-dimensional fluid dynamics the interaction goes through the medium with a flow velocity u_i, pressure p, shear viscosity η and bulk viscosity ζ. The stress tensor is written as

$$T_{ij} = \delta_{ij}p - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right) - \zeta\delta_{ij}\partial_k u_k. \quad (5.1)$$

where i, j, k run the three-dimensional space only. It describes a plasma slightly out of equilibrium, and therefore the temperature and the mean velocity vary in space. We have imposed the condition $T_{0i} = 0$ in the local rest frame, where 0 is the time direction. For the $\mathcal{N}= 4$ SYM theory the energy-momentum tensor is traceless $T_\mu{}^\mu = 0$. Therefore, we obtain the energy density as $\epsilon \equiv T_{00} \equiv 3p$, and the bulk viscosity identically vanishes $\zeta = 0$. The shear viscosity is obtained from correlators of the stress tensors by computing the Green functions.

5.2 Non-Abelian gauge field theory and QCD

From experimental results the wave functions of quarks are known to have the symmetry of gauge group, which is non-Abelian. Therefore, QCD, the theory which describes dynamics of quarks, is SU(3) non-Abelian gauge field theory, where the number 3 comes from the number of colors.

On the other hand, the color symmetry in the string theory is determined by the number of D-branes which stack on three-dimensional space. The end points of open strings, which represent gauge particles, should be always on one of the D-branes. The combinatorics tells us that the gauge group should be SU(N) group for N D-branes. Therefore the setup for QCD, an SU(3) gauge theory, is to have three D-branes. However, to make use of the duality to the string theory we need a large number of D-branes and take a large N limit. In this limit open strings do not affect the position of the D-branes by their vibrations.

5.3 Drag force from the AdS/CFT correspondence

The string theory describes particles in the string picture rather than in the point particle picture. The action of a string is

$$S = \int \left[d^2\sigma \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\nu + \cdots \right], \tag{5.2}$$

where a, b run the worldsheet, the position of the string, μ, ν run the target spacetime, in which the string is embedded. X^μ denote bosonic degrees of freedom of the string, and dots represent terms depending on fermionic degrees of freedom.

The string theory exists now in spacetime in the presence of large N D-branes. However, it is difficult to construct the string theory on a compactified manifold in general, especially the formulation so far has not been well established. In the Green-Schwarz formalism the action in the generic background is known for twenty years, but a concrete expression for a specific background was rarely known. One of the partially known backgrounds is AdS_5 space [18, 19]. According to the AdS/CFT conjecture the geometry of a bunch of D-branes gives AdS_5 by taking the near horizon limit. Therefore, it is a good challenge to think about physics brought by the string theory on AdS_5 space.

5.4 String duality

Gubser conjectured recently [20] that the drag force has an interpretation in the string theory compactified on AdS_5. It is motivated by the previous discussions of the micro state counting of black holes in the string theory. A problem in QGP is that the non-equilibrium thermodynamics is not well described by the quantum field theory. It looks like the information loss problem of black holes [21, 22].

Black hole is well described by the approach used in the thermodynamics. The main idea is that the information of quanta absorbed by the black hole is not counted by increasing the mass but by the area. Once one admits the area law for the entropy, the black hole entropy agrees with the information entropy. We call such a kind of similarity in two different theories as the duality. The naming was often used after the second revolution of the string theory, mainly for field theories, and then in quite recent years for conformal field theories and gravity.

5.5 Lagrangian and the drag force

Drag force is thought to explain a phenomena of the jet quenching. This is a phenomenon that a charged particle loses its energy as it goes through the QGP. It is difficult to describe the jet quenching in the weakly coupled QCD. However, the fluid dynamics is believed to describe the non-thermal state.

The AdS/CFT correspondence conjectured that the dual gravity in five dimensions describes the strong coupling regime of a field theory. Thus, the fourdimensional $\mathcal{N}$ = 4 SYM physics has been studied in terms of the geometry of AdS_5. By using this correspondence it is attempted to derive the drag force from the string theory.

The geometry of AdS_5 blackhole is obtained from the D3-brane geometry

$$ds^2 = H^{-1/2}(-hdt^2 + d\mathbf{x}^2) + H^{1/2}\left(\frac{dr^2}{h} + d\Omega_5^2\right). \tag{5.3}$$

where $\mathbf{x} = (x, y, z)$ are the spatial coordinates along which the D3-branes extend and

$d\Omega_5$ is the metric of five-sphere S^5 with unit radius. H, h are harmonic functions

$$H = 1 + \frac{L^4}{r^4}, \quad h = 1 - \frac{r_H^2}{r^4} \tag{5.4}$$

In the near horizon limit we obtain the metric of AdS_5 blackhole

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \frac{r^2}{L^2}(-hdt^2 + dx^2) + \frac{L^2}{r^2}\frac{dr^2}{h} \tag{5.5}$$

times five-sphere with a radius L.

String in this geometry is described by the Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma e^{\phi/2}\sqrt{-\det g_{ab}}, \quad g_{ab} \equiv G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu. \tag{5.6}$$

The equation of motion is

$$\nabla_a P^a{}_\mu = 0, \quad P^a{}_\mu \equiv -\frac{1}{2\pi\alpha'}G_{\mu\nu}\partial^a X^\nu, \tag{5.7}$$

where ∇_a is the covariant derivative with respect to g_{ab}. Taking the static gauge $\sigma^a = (t,r)$ the Lagrangian becomes

$$S=-\frac{1}{2\pi\alpha'}\int dtdr\,,\quad \mathcal{L}=-\sqrt{1+\frac{h}{H}x'^2-\frac{\dot{x}^2}{h}}\,, \tag{5.8}$$

where $\dot{x}$, x' are the time derivative and space derivative in the worldsheet, respectively.

We assume that the string coordinate takes a form

$$x^I(t,r) = vt + \xi(r) + \cdots, \tag{5.9}$$

which means a steady state behavior is achieved at late time. From the Lagrangian with the above form of the string coordinate we define conjugate momentum

$$\pi_\xi \equiv \frac{\partial \mathcal{L}}{\partial \xi'}\,. \tag{5.10}$$

Solving this equation for ξ' we obtain

$$\xi' = \pm\pi_\xi \frac{H}{h}\sqrt{\frac{h-v^2}{h-\pi_\xi^2 H}}. \tag{5.11}$$

To remove the sign ambiguity we assume that a string trails behind the external quark, therefore ξ is positive as π_ξ is chosen to be positive. To avoid imaginary square root in eq. (5.11) we need the condition

$$\pi_\xi \equiv \frac{v}{1-v^2}\frac{r^H}{L^2} \tag{5.12}$$

Solving eq. (5.12) for ξ we obtain

$$\xi = -\frac{L^2}{2r_H}v\left(\tan^{-1}\frac{r}{r_H} + \log\sqrt{\frac{r+r_H}{r-r_H}}\right). \tag{5.13}$$

Let us calculate the flow of momentum from the external quark into infrared dissipation. It is given by

$$\Delta P_I = \int_I dt \sqrt{-g} P^r{}_x = \frac{dp_I}{dt} \Delta t, \tag{5.14}$$

where the integral is over some interval I of length Δt. The radius should not affect the integral as $P^a{}_\mu$ is conserved. This means actually the force is negative, and the drag force is born from it. The drag force is therefore obtained as

$$\frac{dp_I}{dt} = \sqrt{-g} P^r{}_{x^I} = -\frac{\sqrt{1-v^2}}{2\pi\alpha'} G_{x^I} v g^{ra} \partial_a X^v = -\frac{1}{2\pi\alpha'} \frac{r_H^2}{L^2} \frac{v}{\sqrt{1-v^2}}. \tag{5.15}$$

From the definitions of the radius and the temperature of the geometry

$$L^4 \equiv g_{\text{YM}}^2 N\alpha'^2, \qquad \text{T} = \frac{r_H}{\pi L^2} \tag{5.16}$$

we obtain

$$\frac{dp_I}{dt} = -\frac{\pi\sqrt{g_{\text{YM}}^2 N}}{2} T^2 \frac{v}{\sqrt{1-v^2}}. \tag{5.17}$$

The last factor can be expressed in terms of the external quark momentum and mass as

$$\frac{v}{\sqrt{1-v^2}} = \frac{p_I}{m} \tag{5.18}$$

Then, eq. (5.17) can be solved as

$$p_I(t) = p_I(0)\, exp\left(-\frac{t}{t_o}\right), \qquad t_o = \frac{2}{\pi\sqrt{g_{\text{YM}}^2 N}} \frac{m}{T^2} \tag{5.19}$$

6. Deriving the jet quenching parameter

Klebanov-Tseytlin geometry [23] is slightly different from the $AdS_5 \times S^5$, for which the dual gauge theory is unconfined [24]. It is dual to a $\mathcal{N} = 1$ SYM in which confinement is realized. Since the suppression happens only for the Au–Au collision, and not for the Au–d collision, the theory should be in the strong coupling region with full of high dense matter. Therefore, the Klebanov-Tseytlin model is a rather realistic one among the models that use a geometry. The jet quenching parameter is generated by the Coulomb interaction (electromagnetic interaction), and therefore the Bethe-Bloch formula can be applied.

Also, from a theoretical point of view it is interesting to see what role the compactified isometry plays in the real physics; if it looks like the Randall-Sundrum model, then it is another possibility that QGP can show the fifth dimension as well as a new mechanism of hadron generations.

7. Summary and Conclusion

We reviewed how we describe the perturbative picture in terms of the Feynman diagrams, and introduced the recent results from the RHIC experiment, on which we might see any string theoretical effects. The experiment showed that the hadronic matter behaves as a perfect fluid. This result could be described by the $\mathcal{N} = 4$ SYM theory, which is brought by the AdS/CFT correspondence. The drag force between gluons and quarks could be interpreted by a variant of the string action compactified on the AdS_5 geometry.

Although the superstring theory has supersymmetry, we do not see superpartners of the known particles yet at our energy scale. The AdS/CFT correspondence, however, seems to give some answers to the observation at the experiment. A key to solve the puzzles from the AdS/CFT point of view is to extend the string theory on the AdS_5 geometry or to find an exact geometry which is dual to QCD.

The RHIC experiment also gives further insights to the picture of the Early Universe at the transition point from the radiation dominant era to the matter dominant era. It is also interesting to see the transition in the language of the superstring theory.

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